Special two-day course

``Finding minimum cost paths and mean cycles in a digraph''

Dept. of System Design, Tokyo Metropolitan University
organized by Professor Naoyuki Kubota

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Instructor:
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Synopsis:

In this two-day lecture, we discuss the following topics:
(a) Dynamic programming principles for shortest-path problems;
(b) Bellman-Ford (successive approximation) algorithm and its variants;
(c) Minimum mean cycle problems.

We first discuss a fundamental **single-source shortest-path problem** in a digraph with emphasis on the **optimality conditions** based on **Bellman's equations**. In a general digraph including *cycles* and *negative arc costs*, the basic dynamic programming (DP) procedure is well known as the Bellman-Ford algorithm. We then consider the problem of finding a **best walk** that consists of exactly M arcs; we approach this problem by a variant of the Bellman-Ford method. After that, we highlight the **minimum mean cycle problem** in a general digraph. For this challenging problem, we describe two representative approaches: one is based on a variant of the Floyd-Warshall method, a standard approach to the **all-pairs shortest-path problem**, and another is based on a variant of the Bellman-Ford method. In a digraph with n nodes and m arcs, the former method works in O(mn^2), whereas the latter works in O(mn). To understand the latter method, we consider the problem of **zero-cost cycles**; here, it should be noted that the Bellman-Ford method solves the *single-source shortest path problem* successfully in the presence of any zero-cost cycles, although the solution may not be unique. We then explain how the *concept of zero-cost cycles* plays an important role in the development of such an efficient minimum mean-cycle finder. Finally, we shall consider an application to a cyclic staffing problem.