Special two-day lecture:

``Finding paths in general graphs''

at Hino Campus, Tokyo Metropolitan University
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In this lecture, we consider representative shortest-path problems in a general (directed or undirected) graph having cycles. We assume that every arc is associated with cost. The most fundamental problem is to find an optimal path (with no repeated nodes) from source (starting) node to sink (goal) node that minimizes the sum of arc costs. This problem, known as a *single-pair shortest-path* problem, and its many other variants form a large class of shortest-path problems; the dominant concepts for their solution procedures are based on the *dynamic programming principle*. For backward dynamic programming, we define the minimum-cost-to-go value function; then, invoking the *principle of optimality* leads to **Bellman's equations**.

In a general graph, however, Bellman's equations may not be explicit, because there exist unknown quantities on both sides of Bellman's equations due to cycles in a given graph. To resolve this issue, the order of nodes must be determined appropriately. In this lecture, we consider various situations and discuss how to deal with negative-cost cycles and zero-cost cycles.

A great variety of (complex) optimization problems can be solved by procedures equipped with shortest-path algorithms. This lecture would be helpful for students to attack such a practical optimization problem as *robot navigation*.