The Workshop on Topological Intelligence

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Graph and Network Algorithms for Robot Navigation and Path Planning

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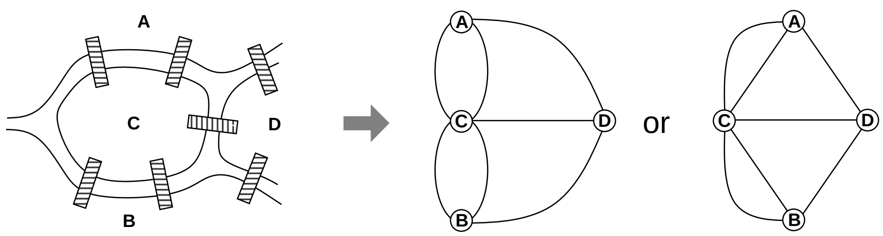
Lecture Synopsis

This invited lecture offers a tutorial on the graph and network algorithms with various illustrative examples mainly for optimization. The conventional wisdom available in the literature of *topology and graph theory* would provide useful information for robot navigation and path planning. Consider, for instance, a celebrated Königsberg bridge problem, whose layout is illustrated in the leftmost figure below: The Pregel River in Königsberg (now called Kaliningrad in Russia) had two banks (labeled A and B), and its splitting forms two islands (labeled C and D). Seven bridges crossed the river, linking those four regions (A, B, C, D). Now, a question below arises:

Question: "Is it possible to make a round trip (or a **tour**), starting on one of the regions, traversing each bridge *exactly once*, and returning back to the starting region?"



The problem can be distilled into a **graph** that consists of four **vertices** (for four *regions* A, B, C, D) and seven **edges** (representing seven *bridges*); two possible graph representations are displayed above right (although both are essentially identical). Then, the above question can be translated as below:

Question (rephrased) "Given the above graph (having four vertices and seven edges), is it possible to start at some vertex, go along *each edge exactly once*, and end up at the starting vertex?"

Below are two relevant problems discussed well in the literature:

Postman's problem:
A postman wishes to find a **tour that traverses each edge only once** and returns to the starting vertex.

Salesman's problem:
A salesman wishes to find a **tour that visits each vertex only once** and returns to the starting vertex.

In graph-theoretic terms, while the postman seeks an **Eulerian circuit**, the salesman looks for a **Hamiltonian cycle**. Here, two terms, *circuit* and *cycle*, are defined differently; the *lecture begins with the introduction of basic graph-theoretic terminologies*.

Furthermore, when the edges in a graph are *weighted*, the most fundamental problem is to find the **shortest path** from a starting point to a destination point. In particular, the above two problems are widely known as the *Chinese postman problem* (for the shortest circuit) and the *traveling salesman problem* (for the shortest cycle); the former can be solved in polynomial time, whereas the latter is strongly NP-hard. In the lecture, we discuss how to approach such representative optimization problems using small examples.